



AN ACOUSTIC DECAY MEASUREMENT BASED ON TIME-FREQUENCY ANALYSIS USING WAVELET TRANSFORM

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In acoustic decay measurement using the third-octave band pass filter, it is known that an inevitable experimental error is produced by "ringing" at the tail part of the impulse response of the third-octave band pass filter. This ringing gives rise to distortion of the decay curve. In order to reduce this error and to obtain an acceptable acoustic decay curve, it has been recommended that the product of the 3 dB bandwidth *B* of the third-octave band pass filter and the reverberation time T_{60} of the room under test be at least 16. For a listening room having short reverberation time and at the low-frequency band with narrow bandwidth, the decay curve cannot, therefore, be measured reliably by using the third-octave band pass filter. In this paper, the continuous wavelet transform (CWT) has been proposed to determine accurately the decay curve with a low value of BT_{60} . The CWT decomposes an acoustic decay signal into time-frequency domain using the third-octave band wavelet filter bank. When the CWT is applied to the measurement of an acoustic decay curve, it is found that the requirement $BT_{60} > 16$ can be replaced by the replacement $BT_{60} > 4$.

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1. INTRODUCTION

An accurate determination of acoustic decay is important for both absorption measurement in a reverberation chamber and the evaluation of concert hall acoustics. It is known that there are two sources of errors in acoustic decay measurement using the third-octave band pass filter, which is the traditional method: smoothing produced by the averaging device and "ringing" of band pass filter [1]. In traditional acoustic decay measurement, the room under test is excited by random noise for a few seconds and then the decay of the sound field is observed on the recorder after the excitation is switched off. Called "the method of interrupted noise" [2], its limited application to room acoustics has long been recognized because the true nature of the decay is often obscured by random fluctuation in the decay curve. In order to minimize the effect of this fluctuation, a number of random noise decay curves are smoothed by an averaging device. In this averaging process, experimental error is produced. To avoid this error, the limitation for the integration time of the averaging device has been required [1, 3]. Alternatively, this experimental error can be solved by using "the method of integrated impulse response" suggested by Schroeder [4]. Along with this error, a more subtle but equally fundamental error is due to the third-octave band pass filter unless the impulse response of the filter is much shorter than that of the room; the resulting acoustic decay curve will be significantly affected. Its basic reason is due to "ringing" at the tail part of the impulse response of the third-octave band pass filter. Kürer and Kurze [5]

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have touched upon this subject. Jacobsen [1, 3] also did a good job in the subject. Jacobsen suggests that for a given band pass filter of bandwidth B, the acceptable acoustic decay curve is obtained only if the requirement, $BT_{60} > 16$, is met, where T_{60} is the reverberation time of the room under test. However, not only can a listening room having short reverberation time not meet this requirement, but at the low-frequency band with narrow bandwidth, it also cannot be satisfied [1, 6, 7]. Reverberation times for both cases cannot, thus, be measured reliably by the traditional method. Therefore, it is important to develop methods to reduce the influence of the filter. In order to reduce the effect of filter, the time-reversed decay technique [3] is suggested; the inequality, $BT_{60} > 16$, can be replaced by the replacement, $BT_{60} > 4$. However, in the time-reversed decay technique, there is an unwanted result, in which the third-octave band pass filter tends to delay the process too much. The purpose of this paper is to demonstrate that the continuous wavelet transform (CWT) can reduce the influence of filter without too much delaying process compared with the reverse-time technique. The time-frequency analysis using wavelet transform is a useful tool to obtain local formations for non-stationary signals. It does not have any cross-term problem occurring in bilinear time-frequency transforms [8]. The CWT decomposes an acoustic decay signal into time-frequency domain using the third-octave band wavelet filter bank [9, 10]. The third-octave band wavelet filter bank is composed of series of the Fourier-transformed version of the mother wavelet. There are many mother wavelets available for the wavelet transform [9]. For the purpose of this paper, the mother wavelet should exponentially decay without "ringing" at the tail of the mother wavelet and its Fourier transformed version should be a constant-Q (quality factor) band-limit. In order to satisfy these two conditions, harmonic wavelet and Morlet wavelet can be considered. The harmonic wavelet is an orthonormal wavelet and Morlet wavelet is a non-orthonormal wavelet. Therefore, the harmonic wavelet has a compactly supported Fourier transform. However, it decays with "ringing" at the tail of the mother wavelet [10]. A critical limit of harmonic wavelet is that, after harmonic wavelet transform, the number of time samples of the time-frequency map is not the same as the number of time samples of an acoustic decay signal. If the number of time samples of an acoustic decay signal is 2^n , that of the time-frequency map transformed by harmonic wavelet is 2^{n-2} . Therefore, the mother wavelet used throughout this paper is the modified Morlet wavelet [11] since its Fourier transformed version can be easily controlled depending on the field of application and does not have "ringing" at the tail of the mother wavelet.

2. THE THIRD-OCTAVE BAND PASS FILTER BANK

The traditional or commercial third-octave band pass filter [12] being used for the measurement of acoustic decay curve is the third-octave band pass filter recommended by ANSI1.11-1986 [13] and IEC225-1966 [14]. The bandwidths *B* of these filters are the same but filter shapes are different, each having different slopes in dB per octave. In many applications it is desirable that the third-octave band pass filter is as selective as possible. However, the attenuation slope of the filter also significantly affects the measurement of the decay curve with low value of BT_{60} [1]. In section 4, the influence of the third-octave band pass filter used throughout this paper is the Order 3 Butterworth filter designed using MATLAB [15], which is a family of Class III third-octave band pass filters recommend by ANSI1.11-1986 [13]. Its attenuation slope is around 50 dB per octave. Hence, the filter Order designation is directly correlated with the number of resonators and pole pairs in an analog filter design including the prototype for an infinite impulse response or recursive digital filter [13]. Figure 1 shows



Figure 1. The third-octave band pass filter bank.

a constant-Q (quality factor) third-octave band pass filter bank, which is going to be used throughout this paper.

3. THE THIRD-OCTAVE BAND WAVELET FILTER BANK

The continuous wavelet transform [16] is based upon a family of functions,

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right), \quad a > b \in \Re$$
(1)

where ψ is a fixed function, called the "mother wavelet," that is localized both in time and frequency. The function $\psi_{a,b}(t)$ is obtained by applying the operations of shifting (*b*-translation) in the time domain and scaling in the frequency domain (*a*-dilation) to the mother wavelet. The mother wavelet used throughout this paper is the Morlet wavelet [17],

$$\psi(t) = \frac{1}{\sqrt{\pi B}} e^{j\omega_0 t - (t^2/B)},$$
(2)

where ω_0 is the center frequency of the "mother wavelet" and *B* is the bandwidth defined as the variances of the Fourier transform $\Psi(f)$ of the Morlet wavelet

$$B = \int_{-4}^{+4} f^2 \Psi^*(f) df.$$
 (3)

The CWT of a signal x(t) is defined by

$$W_{x}^{\psi}(a,b) = \int_{-4}^{+4} x(t) \,\psi_{a,b}^{*}(t) \,\mathrm{d}t = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \psi^{*}\left(\frac{t-b}{a}\right) \mathrm{d}t,\tag{4}$$

where $\psi^*(\cdot)$ is the complex conjugate of $\psi(\cdot)$ and the function x(t) satisfies the condition,

$$\|x\|^{2} = \int_{-4}^{+4} |x(t)|^{2} dt < +\infty.$$
⁽⁵⁾

Here, $\psi_{a,b}(t)$ plays an analogous role to the $e^{j\omega t}$ in the definition of the Fourier transform, if the mother wavelet, $\psi(t)$, satisfies the admissibility condition:

$$C_{\psi} = \int_{-4}^{+\infty} \frac{|Y(\omega)|^2}{|\omega|} d\omega < +\infty, \tag{6}$$

The inverse wavelet transform can be obtained by

$$x(t) = \frac{1}{C_{\psi}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} W_{x}^{\psi}(a, b) \psi_{a,b}(t) \frac{\mathrm{d}b \,\mathrm{d}a}{a^{2}}.$$
 (7)

An alternative formulation of the CWT, equation (4), can be obtained by expressing x(t) and $\psi(t)$ via their Fourier transforms, $X(\omega)$ and $\Psi(\omega)$, respectively:

$$W_x^{\psi}(a,b) = \sqrt{a} \int_{-\infty}^{+\infty} X(\omega) \Psi^*(a\omega) e^{j\omega b} d\omega.$$
(8)

The relationship between the scale parameter a and frequency ω can be expressed as [18, 19]

$$a = \frac{\omega_0}{\omega},\tag{9}$$

where ω_0 is the center frequency of the "mother wavelet" (see equation (2)). From a signal processing point of view, a wavelet basis generates a constant-Q octave-band or octave band filter bank structure [9, 10]. Therefore, the third-octave band filter bank can be generated. It is called "the third-octave wavelet filter bank" throughout this paper and is plotted in Figure 2. In this graph, the logarithmic scale of the co-ordinate allows us to view the constant-Q characteristics of the wavelet basis more clearly. The design of the



Figure 2. The third-octave band wavelet filter bank.

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third-octave wavelet filter bank and its influence on the decay curve are discussed in section 4 in detail.

4. INFLUENCE OF THE FILTER BANKS

In sections 2 and 3, two different types of filter banks were discussed: the third-octave band pass filter and the third-octave band wavelet filter. The influence of the third-octave band pass filter on the measurement of acoustic decay curve has been greatly well discussed by Jacobsen [1]. According to Jacobsen's research results, a distortion of the energy decay curve determined in the frequency band occurs unless the impulse response of the filter is much shorter than the impulse response of the room under test [3]. Therefore, to obtain an accurate decay curve, for a given third-octave band pass filter of bandwidth B and reverberation T_{60} of the acoustic room, the following requirement

$$BT_{60} > 16$$
 (10)

should be met. Figure 3 shows these results. In this graph, the function used as a decay curve of the room under test is the following ideal decay function.

$$g_i(t) = \begin{cases} e^{t/2\tau} \cos \omega_b t & \text{for } t > 0, \\ 0 & \text{elsewhere,} \end{cases}$$
(11)

where ω_b is 125 Hz and the τ is the decay time and is related to reverberation time $T_{60} = 6\ln(10)\tau$ [1]. The influence of the third-octave band pass filter under idealized conditions can be calculated by convolving the impulse responses $h_b(t)$ of the third-octave band pass filter with the ideal decay function $g_i(t)$. The impulse response of the combination of filter and room is

$$g_m(t) = \int_{-\infty}^{\infty} g_i(\lambda) h_b(t-\lambda) d\lambda.$$
 (12)

The decay curve is calculated by integrating the squared function [4],

$$d(t) = \int_{t}^{4} g_{m}^{2}(\lambda) \,\mathrm{d}\lambda, \tag{13}$$

Ideally, if the ideal decay function $g_i(t)$ is not affected by filter, the impulse response $g_m(t)$ of the combination of filter and room becomes the ideal decay function by itself. The decay curve d(t) is calculated directly related to the envelope of the ideal decay function. Therefore, the decay curve can be obtained by substituting the envelope of the ideal decay function into equation (13).

$$d(t) = \int_{t}^{\infty} e^{\lambda/\tau} d\lambda$$
 (14)

and the logarithmic value of decay curve d(t) is given by

$$10\log_{10} d(t) = -\alpha t,$$
 (15)

where α is the slope rate of decay and function of τ . Since $T_{60} = 6\ln(10)\tau$, the only slope rate of the decay curve depends on T_{60} and the shape of slope of decay curve is a straight line. In practice, the ideal decay function $g_i(t)$ is affected by the filter. Therefore, the impulse response $g_m(t)$ of the combination of filter and room is calculated by convolving the ideal decay function with the impulse response $h_b(t)$ of the third-octave band pass filter with bandwidths *B*. The decay curve can be obtained by substituting this result into equation (13). Its mathematic formula is given by

$$d(t) = \int_{t}^{\infty} \left(\int_{-\infty}^{\infty} e^{3\lambda \ln(10)/T_{60}} \cos \omega_b \lambda (h_b(\eta - \lambda)) d\lambda \right)^2 d\eta,$$
(16)

where $h_{h}(t)$ is the impulse response of the third-octave band pass filter with bandwidths B. The impulse response of the third-octave band pass filter has various shapes depending on bandwidth B and the number of poles. This impulse response is generally obtained by taking the inverse Fourier transform of the frequency-response function of the IIR filter designed in the frequency domain. Its typical shape used throughout this paper is shown in Figure 4 and designed by using MATLAB [15]. Since equation (16) is very complex depending on the impulse-response function of the third-octave band pass filter with bandwidths B, it is often difficult to calculate the decay curve analytically. In this paper, the decay curves are calculated numerically depending on the various values of BT_{60} and plotted in Figure 3. As mentioned in equation (15), if an ideal decay function is not affected by the filter, the decay curve becomes a straight line. However, it is known that the decay curves at 5 dB below stationary level are distorted as shown in Figure 3 when the value of BT_{60} is not at least 16. Therefore, it can be understood that the decay curves are seriously affected by a filter. Although the value of BT_{60} is larger than 16, the initial parts of decay curves at 1 dB below stationary level are still distorted. The distortion is due to the "ringing" at the tail part of the impulse response of the third-octave band pass filter as shown in Figure 4. If the room has a double-sloped decay characteristic, the initial decay rate is important for the determination of the statistical absorption coefficient of the test material [4]. In order to determine the initial part of decay, a short interval beginning at 1 dB below the stationary level should be an adequate requirement. To satisfy this requirement, the following strong requirement,

$$BT_{60} > 64$$
 (17)

should be met [1]. These influences of the third-octave band pass filter, i.e., the distortion of the decay curve, can be reduced by the third-octave band wavelet filter based on the CWT

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Figure 4. Impulse response of the third-octave band pass filter with the center frequency 32 Hz.

using the Morlet wavelet. The third-octave band wavelet filter bank is plotted as shown in Figure 2 and is obtained by calculating the 3 dB bandwidth *B* and the center frequency ω_i using equations (3) and (9). First, the center frequency of the third-octave band wavelet filter is determined by applying the operation of scale parameter *a* to equation (9). The center frequency ω_i is, therefore, given by

$$\omega_i = \frac{\omega_0}{a_i} \tag{18}$$

where $a_i = 2^{i/3}$, i = 1, 2, 3, 4, ..., n. Second, the 3 dB bandwidth B, the variances of the Fourier transform $\Psi(f)$ of the Morlet wavelet, is calculated by equation (3) and is also dilated and constricted by the value of the scale parameter a_i . The shape of the third-octave band wavelet filter is compared with the shape of the third-octave band pass filter in frequency domain as shown in Figure 5. The shapes of both filters are within the range recommended by ANSI Class III except that the shape of the wavelet filter is slightly out of range as shown in Figure 5(b) in detail. The attenuation slopes of both filters are different from each other as shown in Figure 5(b) in detail. A different attenuation slope yields a different impulse response. The impulse response of the third-octave band wavelet filter is the dilated or constricted Morlet wavelet. Figure 6 shows the Morlet wavelet. The Morlet wavelet is the modulated Gaussian function as explained in equation (2). The Morlet wavelet has no "ringing" at the tail part since its envelope decays exponentially. The effect of the third-octave band wavelet filter in acoustic decay measurement is illustrated in Figure 7. The ideal decay function used for this test is the same as the function given in equation (11). The wavelet transform for the ideal decay function is obtained by substituting equations (11) and (2) into equation (4)

$$W_{x}^{\psi}(a,b) = \int_{-\infty}^{+\infty} g_{i}(t) \psi_{a,b}^{*}(t) dt = \frac{1}{\sqrt{a}} \int_{0}^{+\infty} \frac{1}{\sqrt{\pi B}} \exp\left(j(t-b)\left(\frac{\omega_{0}-\omega_{b}+1/2\tau}{a}\right) - \frac{1}{B}\left(\frac{t-b}{a}\right)^{2} dt.$$
(19)



Figure 5. Response curves for third-octave band filters shapes in frequency domain: (a) dynamic range 80 dB (b) dynamic range 23 dB.

At the interesting center frequency ω_i , the impulse response $g_m(t)$ of the combination of wavelet filter and ideal decay function $g_i(t)$ is obtained by slicing through the wavelet transformed version $W_x^{\psi}(a, b)$ along time shift b at the scale parameter $a = \omega_0/\omega_i$. Finally, the energy level of the decay curve is calculated by substituting $g_m(t)$ into equation (13). According to these results, when the third-octave band wavelet filter bank is applied, if the BT_{60} is larger than 4, the acoustic decays are nearly straight-lines at 5 dB below stationary level, although their initial parts at 5 dB below stationary level are still distorted until the value of BT_{60} is at least 16. Therefore, when the CWT is used, a new requirement to obtain the acceptable decay curve is suggested so that the value of the BT_{60} is at least 4. If the initial part distortion is considered as the important point of view, when the third-octave band wavelet filter is used, the strong requirement, $BT_{60} > 64$ is suggested in equation (17). However, when the third-octave band wavelet filter is used this inequality $BT_{60} > 64$ can be replaced by the inequality $BT_{60} > 16$. Figure 8 illustrates these results. Both decay curves



Figure 6. Impulse response of the third-octave band wavelet filter with the center frequency 32 Hz (i.e., Morlet wavelet scaled by scaling parameter *a*).



Figure 7. Acoustic decay curves calculated by applying the third-octave band wavelet filter bank to the exponential decay function oscillating with the frequency 125 Hz; ----, $BT_{60} = 4$, -----, $BT_{60} = 8$; -----, $BT_{60} = 32$.

are straight lines at 1 dB below stationary level, but the values of BT_{60} are different from each other. The value of BT_{60} for the decay obtained using the third-octave band pass filter is 64 while the value of BT_{60} for the decay obtained by using the third-octave band wavelet filter is 16. Figure 9 shows the comparison between the decay curves obtained by using the third-octave band pass filter bank and the decay curves obtained by using the third-octave band wavelet filter bank. The ideal decay function used for this test is given by

$$g_i(t) = \begin{cases} e^{t/2\tau} \cos \omega_i t & \text{for } t > 0, \\ 0 & \text{elsewhere} \end{cases}$$
(20)



Figure 8. Comparison of the initial part of the acoustic decay curves calculated by applying the third-octave band filter banks to the exponential decay function oscillating with the frequency 125 Hz, -----, by using the third-octave band wavelet filter with $BT_{60} = 16$; -----, by using the third-octave band pass filter with $BT_{60} = 64$.

where ω_i is the center frequency of the third-octave band defined in equation (18). When the third-octave band pass filter is used, the acceptable decay curve cannot be obtained until the value of BT_{60} is larger than 15 as shown in Figure 9(d). The reverberation time T_{60} used under test is 1 s. However, when the third-octave band wavelet filter is used, an acceptable decay curve is obtained under the condition that the value of BT_{60} is larger than 4.6 as shown in Figure 9(a) and 9(b). In particular, the distortion at the initial part of decay curve obtained by the third order third-octave band pass filter is serious until the value of the BT_{60} is around 59, as shown in Figure 9(f). This value of BT_{60} corresponds reasonably to the inequality, $BT_{60} > 64$, required in equation (17). When the third-octave band wavelet filter is applied, the distortion at the initial part of decay curve hardly occurs under the condition that the value of BT_{60} is larger than 15, as shown in Figure 9(d). Therefore, if the volume of the room under test is small or has high-absorption material, the reverberation time of the room becomes short. In this case, the third-octave wavelet band filter bank is a very useful method, instead of the third-octave band pass filter bank, to obtain accurate determination of the reverberation time. As a conclusion, the value of the BT_{60} required to obtain an acceptable decay curve is listed in Table 1. Finally, if the value of BT_{60} is very large, then the influences of the both filters are not significant as shown in Figure 10(a) and 10(b). At or above the center frequency 1000 Hz, the decay curves are nearly the same as the ideal decay function given in equation (20).

5. CONCLUSIONS

The traditional third-octave band pass filter has been used for long time to obtain the acoustic decay curve. However, this acoustic decay curve is distorted by the influence of the third-octave band pass filter. This distortion is basically due to "ringing" at the tail part of the impulse response of the third-octave band pass filter. To obtain an acceptable decay curve using the third-octave band pass filter, the value of the BT_{60} should be at least 16. In this paper, in order to reduce this distortion phenomenon, a new method based on the



Figure 9. Comparison between the decay curves obtained by using the third-octave band pass filer bank and the decay curves obtained by using the third-octave wavelet filter bank when T_{60} is 1., the third-octave band pass filter,:, the third-octave band wavelet filter; ----, the ideal decay function. (a) $BT_{60} = 3.7$ and the center frequency 16 Hz; (b) $BT_{60} = 4.6$ and the center frequency 20 Hz; (c) $BT_{60} = 7$ and the center frequency 32 Hz; (d) $BT_{60} = 15$ and the center frequency 64 Hz; (e) $BT_{60} = 30$ and the center frequency 128 Hz; (f) $BT_{60} = 59$ and the center frequency 256 Hz.

continuous band wavelet transform (CWT) is suggested. The CWT is composed of the third-octave band wavelet filter bank. The third-octave wavelet filter uses the modified Morlet wavelet as its impulse response. The Morlet wavelet is a modulated Gaussian function. Its envelope decays exponentially and has no "ringing" at the tail part. In this



Figure 10. Comparison between the decay curves obtained by using the third-octave band pass filter bank and the decay curves obtained by using the third-octave wavelet filter bank; when T_{60} is 1., the third-octave band pass filter;, the third-octave band wavelet filter;, the ideal decay function. (a) $BT_{60} = 119$ and the center frequency 500 Hz; (b) $BT_{60} = 237$ and the center frequency 1000 Hz.

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Comparison of the values of the BT_{60} required for obtaining acceptable decay curve by the third-octave filter banks

| Filter Bank Type | Straight decay line at 5 dB below stationary level | Straight decay line at 1dB below stationary level |
|-----------------------|--|---|
| Band pass filter bank | $BT_{60} > 16$ | $BT_{60} > 64$ |
| Wavelet filter bank | $BT_{60} > 4$ | $BT_{60} > 16$ |

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method, the value of the BT_{60} required to obtain the acceptable decay curve is at least 4 instead of 16. If the initial decay is important, the value of the BT_{60} should be larger than 16. Therefore, the wavelet filter bank is a useful tool for accurately determining the acoustic decay curve with low value of the BT_{60} .

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